*DS bullet problem in Puzzles for Programmers ---15/6/2018*

*Consider a 6 chambered circular configuration of bullets and 2 bullets are loaded in adjacent chambers. The enemy fires but an empty slot fires and you are still alive. Before he fires a second time, gives you 2 options. Spin the chamber or click again. Which option would you choose?*

*The various configurations where bullets can be stored side by side are as under:*

*Go in counter clockwise direction with the bullet firing from slot 1*

*-------------> anticlockwise direction*

*B B E E E E (B = Bullet and E = empty slot)*

*1 2 3 4 5 6 [1]*

*E B B E E E [2]*

*1 2 3 4 5 6*

*E E B B E E [3]*

*1 2 3 4 5 6*

*E E E B B E [4]*

*1 2 3 4 5 6*

*E E E E B B [5]*

*1 2 3 4 5 6*

*B E E E E B [6]*

*1 2 3 4 5 6*

*Of all the 6 configs of bullets in the slots:*

*If a spinning is undertaken, the revolver will end up in any of the above configs.*

*Bullet will fire from slot 1 and hence favorable configs and hence P(D) = prob of death = 2/6 = 1/3*

*Therefor P(S) = prob of survival = 1 - P(D) = 1 - 1/3 = 2/3*

*If survived after one click, then we need to eliminate the configurations where Bullet was in slot 1 and this leads us to having the sample space as 4 configurations [2],[3],[4] and [5]. Of these the config leading to bullet firing if clicked again is [5]*

*hence P(Death\_after\_first\_click) = 1 / 4*

*Hence P(Survival\_after\_first\_click) = 1-(1/4) = 3/4*

*Now P(survival\_after\_first\_click) > P(S) = 3/4 > 2/3*

*Hence in order to survive, the revolver must not be spun after first click*

*Now suppose the trigger is pulled two more times for a total of 4.*

*The configurations if one is still alive after the second trigger pull are*

*E E B B E E*

*1 2 3 4 5 6*

*E E E B B E*

*1 2 3 4 5 6*

*E E E E B B*

*1 2 3 4 5 6*

*Would it be advantageous to ask for a spin before the third trigger pull?*

*No Spin P(D) = 1/3 ( from the configs above) = P(D) = 1/3 ( Spin undertaken)*

*Adv of spinning now and surviving the 3rd trigger pull will increase survival prob for 4th trigger pull = 3/4*

*Using this strategy, (No spin before 2nd trigger pull, spin before the 3rd, No spin before 4th pull) chances of survival post the first trigger pull*

*= (3/4)\*(2/3)\*(3/4) = 3/8*

*However if spun every time P(S) = 2/3 \* 2/3 \* 2/3 = 8/27*

*3/8 > 8/27 ==> 27\*3 > 64 which is true*

*Hence the strategy of No Spin, spin, No spin works best. Lovely!*

*2] Design series of cuts to partition a square pie into equal pieces to share among several pieces*

*Piece offered to each person is called final piece.*

*Rules:*

*1] All cuts must be straight, vertical and parallel to one of the original sides*

*2] All final pieces must obtain the same amount of pie volume (and because of the first rule) the same amount of the top of the pie*

*3] Every cut should yield one final piece except the last one, which yields two final pieces.*

*Consider a square (a unit one) and orient it along the coordinate axes so that its coordinates are 0,0 1,0 , 1,1 and 0,1*

*Make 2 cuts along the Y axis parallel to each other. (3 pieces of equal size are generated)*

*Now in the second case, make one parallel cut along the Y axis and one cut along the X axis. Therefore 2 perpendicular cuts.*

*They both satisfy the rules.*

*However the key question is: which has the smaller total perimeter?*

*In Case 1, the perimeter = 1 + 1/3 + 1 + 1/3 = 8/3*

*So the 3 final pieces have a cumulative perimeter of 8/3 \* 3 = 8*

*Two perpendicular cuts yield one piece of 8/3 and 2 final pieces each having 2/3 + 1/2 + 2/3 + 1/2 = 7/3.*

*Here the total perimeter = 14/3 + 8/3 = 22/3*

*Thus the second case minimises the total perimeter.*

*How can we keep the constraints intact and get 5 pieces?*

*Perimeter should be minimal*

*Along the Y axis, make 2 parallel cuts, one along the X axis along the length of the cake such that cut is parallel to X axis and finally Perpendicular to the X axis along the middle. That’s 4 cuts in total to yield 5 pieces.( last cut yields 2 pieces so constraints are met)*

*There are 3 pieces generated after the first 3 cuts---> Perimeter of one such piece = 1/3 + 1/2 + 1/3 + 1/2 = 1 + 2/3 = 5/3*

*Therefore total perimeter of 3 pieces = 3\*(5/3) = 5*

*Now, the last 2 pieces yield 2 pieces each of p = 1/2 + 1/2 + 1/2 + 1/2 = 2*

*Total p here = 4*

*Therefore final total p = 9 in this case*

*Case 2*

*Make 2 parallel cuts to get 2 rectangular equal pieces and 2 final pieces by one perpendicular cut*

*P of the 2 rectangular pieces = 2 \* ( 1 + 1/3 + 1 + 1/3) = 2 \* ( 2 + 2/3) = 2\*(8/3) = 16/3*

*There are remaining 2 pieces whose total p = 2 \*(1/2 + 1/3 + 1/2 + 1/3) = 2\*(1 + 2/3) = 2\*(5/3) = 10/3*

*Final Total p = 26/3*

*9 > 26/3 as 27 > 26*

*Hence the second case is better as it minimises the p by 1*

*If we eliminate the last rule, the last cut need not yield 2 final pieces:*

*Make 2 parallel cuts and then 2 perpendicular cuts to get 4 equal rectangular pieces and one final rectangular piece that makes a total of 5 pieces.*

*Final total p = (1/2 + 1/3 + 1/2 + 1/3)\* 4 + ( 1 + 1/3 + 1 + 1/3) = (5/3)\*4 + (8/3) = 20/3 + 8/3 = 28/3*

*Make 2 parallel cuts along X axis and 2 perpendicular cuts along Y leading to 4 + 1 = 5 cakes*

*Total final p = 4 \*(5/3) + 8/3 = 20/3 + 8/3 = 28/3*

*Not really a different configuration as can be obtained by flipping the configuration above*

*One parallel cut, and 3 perpendicular cuts to yield 5 pieces total*

*Final total p of all pieces = (1/3 + 1 + 1/3 + 1) + (2/3 + 1/4 + 2/3 + 1/4) \* 4 = 8/3 + 11/6 = 9/2*

*9/2 < 28/3 and hence this configuration yields the min possible perimeter.*